Is the flat spacetime related to a kinematical structure?

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Snyder introduced quantized spacetime in 1947. As an example, the commutator of position momentum ($[x, p] = i\hbar$) is represented as (Snyder, 1947):

$$[x, p_x] = i\hbar\left[1 + \left(\frac{a}{\hbar}\right)^2 px^2\right], \tag{1}$$

Where a is a natural unit of length. The standard commutator is restored when $a \to 0$ .Within this frame, the Einstein Hamiltonian of a lattice of Planck length (λ_P) may be (Glinka,2010):

$$E^2 = c^2p^2 + m^2c^4 + \alpha(\frac{c}{\hbar})^2 \lambda_P{}^2 p^4, \tag{2}$$

where α is a dimensionless constant. Then with linearization Glinka modified Dirac equation to include λ_P. However, Planck length and Planck time are constants and are based on the universal physical constants, according to Planck units system. In other words, they are not related to a physical theory (Meschini, 2006), in other words, they are introduced via dimensional analysis technique.

In all proposed theories, those include Planck length or any unit of length ($\lambda \equiv a$ or λ_P ....) the ordinary spacetime continuum arises as $\lambda \to 0$. This λ works as a boundary between two different worlds.

In 2016, an attempt looked for a possible mathematical derivation of Dirac equation form without aid of quantum postulates and Dirac linearized Hamiltonian (Sanduk, 2017). That approach attributed Dirac equation form to time differentiation of a complex vector:

$$\boldsymbol{\mathcal{Z}} = \boldsymbol{b} \exp i(\boldsymbol{k} \cdot \boldsymbol{r} - \omega t) = b\hat{\boldsymbol{e}}_r \exp i\,(\boldsymbol{k} \cdot \boldsymbol{x} - \omega t)\,. \tag{3}$$

Where $\boldsymbol{b}$ is a real radial vector. The complex vector ($\boldsymbol{\mathcal{Z}}$) is deduced from Dirac Hamiltonian. The obtained derived Dirac equation form postulates is:

$$i\frac{\partial \boldsymbol{\mathcal{Z}}}{\partial t} = (-i\,c\boldsymbol{A} \cdot \boldsymbol{\nabla} + B\omega)\boldsymbol{\mathcal{Z}}\,, \tag{4}$$

Where $\boldsymbol{A} = \mp\, i\,\hat{\boldsymbol{e}}_\theta$ and $B = \pm 1$. Equation (4) describes the evolution process of the function $\boldsymbol{\mathcal{Z}}$. Then, the quantity inside brackets of equation (4) has the form of Dirac Hamiltonian. This equation is for a motion in a complex plane. The second time derivative of equation (4) leads to an equation similar to the Klein-Gordon equation.

$$\frac{\partial^2 \boldsymbol{\mathcal{Z}}}{\partial t^2} = (-c^2\boldsymbol{\nabla}^2 + \omega^2)\boldsymbol{\mathcal{Z}}\,. \tag{5}$$

The quantity inside brackets of equation (5) has the form of the energy-momentum relation of flat spacetime.

The structure of complex vector (equation (3)) can be considered with aid of Euler's formula in a trigonometrical form (Sanduk, 2017).

$$\boldsymbol{\mathcal{Z}}(x,t) = \boldsymbol{b}\,\{\cos\,(\boldsymbol{k} \cdot \boldsymbol{x} - \omega t) + \sqrt{-sin^2(\boldsymbol{k} \cdot \boldsymbol{x} - \omega t)}\} \tag{6}$$

That leads to assume a general real algebraic form for the function 6:

$$\boldsymbol{r} = \boldsymbol{\mathscr{b}}\left\{\cos\,(\boldsymbol{k}_2 \cdot \boldsymbol{s} - \omega t) \pm \sqrt{-sin^2(\boldsymbol{k}_2 \cdot \boldsymbol{s} - \omega t) + X}\right\} \quad . \tag{7}$$

Where X a real dimensionless quantity. This form is for position vector of a point in system of two rolling circles of radii a_1 anda_2 . In addition to that $a_1 \ll a_2$, $a_2 = 1/k_2$, $\mathscr{b} = a_1 + a_2$ and $X = a_1/\mathscr{b}$.
In case of partial observation (Sanduk, 2012) of the system, the quantities $X = 0$ or $a_1 = 0$. That means the small circle can not be recognised. Then equation (7) transforms to equation (6 or 3).
Time differentiation of equation (7):

$$\frac{\partial \boldsymbol{r}(r,t,X)}{\partial t} = \frac{\partial(\boldsymbol{a}_2 + \boldsymbol{\mathscr{b}}\sqrt{X})}{\partial t}\left\{cos(\boldsymbol{k}_2 \cdot \boldsymbol{s} - \omega t) \pm \sqrt{-sin^2(\boldsymbol{k}_2 \cdot \boldsymbol{s} - \omega t) + X}\right\} + (\boldsymbol{a}_2 + \boldsymbol{\mathscr{b}}\sqrt{X})\left\{\omega\ sin(\boldsymbol{k}_2 \cdot \boldsymbol{s} - \omega t) \pm \frac{\omega\ sin(\boldsymbol{k}_2 \cdot \boldsymbol{s} - \omega t) cos(\boldsymbol{k}_2 \cdot \boldsymbol{s} - \omega t) + \frac{\partial X}{\partial t}}{\sqrt{-sin^2(\boldsymbol{k}_2 \cdot \boldsymbol{s} - \omega t) + X}}\right\} \quad .(8)$$

This form does not show any relationship with Dirac equation form, but Dirac equation and Hamiltonian forms (equation (4)) can be restored when $X = 0$. Equation (8) does not show a relativistic Hamiltonian.

Conclusions

1- The space and time combination terms in (4, and 5) are owing to the structure of two rolling circles.

2- The minimum length does not imposed as a limit, but is owing to a generalised kinematical model.
3-The spacetime continuum may be attributed to the effect of the partial observation on the kinematical system of two rolling circles. It looks as the ordinary Lorentzian spacetime arises as $X = 0$. Or $a_1 = 0$, this condition leads to spactime continuum and quantum realm.
4-The imaginary unit vector ($i\ \hat{\boldsymbol{e}}_\theta$) is responsible for the negative square quantity in first terms of equation (5).

5- Both of the spacetime continuum and the complex function arise due to the partial observation.
6- This approach for quantum form shows that there is no need for minimal physical length and time interval as Plank length and time.
7- There is no observable spacetime beyond Minkowski space (microscale).

References

Glinka, L.A. (2010). *Apeiron* 17 (4), 2010, pp. 223-242
Meschini, D., (2006) Planck-scale physics: facts and beliefs, arXiv:gr-qc/0601097v1 23.
Sanduk, M., (2017) Derivation of Dirac equation form using complex vector, to be published in *IOP Journal of Physics: Conference Series.*
Sanduk, M., (2012) A kinematic model for a partially resolved dynamical system in a Euclidean plane, *Journal of Mathematical Modelling and Application* , Vol. 1, No.6, 40-51
Snyder, H.S., (1947a). *Phys. Rev*. **71** 1947, 38-41.