On the fate of "indeterministic" spacetimes beyond classical general relativity

Many general relativistic spacetimes have features which can, arguably, be interpreted as demonstrating indeterminism (in the intuitive sense that state of spacetime at one moment of time does not define a unique state of spacetime at all times) of the theory. Broadly speaking:

1 there can be no globally hyperbolic region at all: either (a) because closed time-like curves exist, or (b) spacetime is strongly causal, but not globally hyperbolic (for example, due to boundary at timelike infinity),

2 globally hyperbolic region is not maximal in the sense that spacetime has been "mutilated" in some way,

3 spacetime is singular — it abruptly "comes to an end", either (a) due to blowup of some quantities, or (b) due to some form of curve incompleteness,

4 spacetime does have maximal globally hyperbolic region which can be extended in multiple ways,

Type 3 is rampant in classical GR (as witnessed by the singularity theorems). But one can see type 3 as demonstrating a breakdown of the theory rather than indeterminism. And even though there are many examples of types 1 and 2, they are sometimes argued to be unphysical at the classical level. This leaves type 4 as most promising for someone who claims that classical general relativity is indeterministic. And indeed such spacetimes can be found (see Chruściel and Isenberg (1993) for relevant technical results concerning existence of such spacetimes). But even if we grant that type 4 demonstrates indeterminism, classical general relativity is not all there is. What happens to such spacetimes beyond classical GR?

I will discuss few different possible ways of removing indeterministic solutions of type 4 in quantum gravity, and comment on the prospects and difficulties they are facing, focusing on:

(a) arguments to the effect that extendible globally hyperbolic spacetimes are not semi-classical solutions (and hence cannot be solutions of the full theory) (see Thorne (1993) for discussion of Misner spacetime in this context),

(b) arguments to the effect that only "causal" spacetimes should be allowed as solutions, since otherwise some mathematical technique gives ill results (for example, some version of "classical limit" cannot be obtained unless non-globally hyperbolic spacetimes are eliminated from suitable path integral),

(c) possibility of showing that in a stochastically generated geometry one almost surely does not end up in a history corresponding to extendible spacetime,

(d) ways in which the requirement of stability of Hawking (1971) can be given foundational significance,

(e) "direct" elimination by applying canonical quantization to the extendible spacetimes.

I will argue that ways (a) and (d) are most promising, but demand (in case (a)) serious attention to the relation between semi-classical and full solutions and (in case (d)) a theorem linking being of measure zero in suitable measure in quantum theory with rarity in topology in which the data are atypical. Moreover, I will suggest that ways (c) and (e) are problematic for approaches which propose that spacetime is fundamentally discrete, since for them to work one needs a well-defined counterpart of the notion of inextendibility for the appropriate type of discrete structures.

References

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